CAPEX-Aware Design of Survivable DWDM Mesh Networks

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***Abstract*—This paper reviews the basic architecture and component costs of opaque, transparent, and semi-transparent DWDM networks and looks at the network design problem from a capital expenditure (CAPEX) point of view. Given are a fiber topology and a demand matrix with different bit rates. Required is the least-cost optical equipment for that topology together with the routing and potential muxponder-based aggregation of all demands such that they can be supported by the newly designed network. We look at the problem for networks without resilience requirements and for survivable networks using 1+1 protection against single fiber cuts. We model this problem for the three types of optical networks by integer linear programs (ILPs) in a canonical way.**

***Index Terms*—Survivable Optical Networks, DWDM, Grooming, Muxponder, 1+1 Protection, CAPEX, ILP**

I. Introduction

New Internet services like IP-TV lead to continuously increasing traffic volumes and analysts even forecast an exponential growth in the future. Thus, Internet service providers (ISPs) must continuously increase the capacity of their networks. Since link and node failures are an inevitable part of daily network operation, ISPs use protection mechanisms to provide high service availability to its users. As the revenues per carried bit decrease, ISPs strive for a cheap, easily extensible, and reliable infrastructure. Optical networks using *dense wavelength-division multiplexing* (DWDM) technology fulfill these requirements. The initial investment is a network of glass fibers between points of presence. Fiber glass is cheap and many fibers are available per fiber bundle. Today, up to 160 wavelengths can be enabled leading to a transmission capacity of several Tbit/s per fiber. Thus, the capacity of such networks seems unlimited. Wavelengths for data transmissions can be incrementally added per fiber link inducing upgrade costs only when needed. This is an economically important technical feature of optical networks.

We consider three types of optical networks: *opaque*, *transparent*, and *semi-transparent* optical networks. In all these networks, demands of various bit rates exist and transponders are used to send the signal of one demand onto one wavelength. Muxponders multiplex the data of several demands of the same bit rate onto a single wavelength. This saves wavelengths and optical transmission equipment. The routing of the demands heavily influences the amount of traffic aggregatable by muxponders and thereby the savings of installation costs. Routing optimization to facilitate efficient data aggregation by muxponders is called *grooming*. Since routing and traffic aggregation have different degrees of freedom in the three considered types of optical networks, grooming has different complexity.

This paper focuses on the installation cost of survivable optical networks. We assume that a demand matrix with different bit rates and a fiber topology are given, but no equipment is installed. The operator needs to determine where the primary and backup paths for the demands are routed and possibly aggregated by multiplexers. Based on this information, the network can be equipped with hardware, in particular with expensive transponders, multiplexers, and port cards. Due to the competitive market, the *capital expenditure* (CAPEX), i.e. the cost for the installed equipment, should be minimized. The contribution of this work is the description of the CAPEX minimization problem for the three considered network types using *integer linear programs* (ILPs). The incremental problem formulation for the three network types and the common nomenclature make their differences explicit and show the different problem complexities.

Sect. II reviews related work regarding optimization of optical networks. Sect. III presents the architecture of opaque, transparent, and semi-transparent DWDM networks from a CAPEX point of view. Sect. IV describes the CAPEX minimization problem for the three network types using ILPs. Sect. V summarizes this work and gives conclusions.

II. Related Work

A general overview of grooming mechanisms in WDM ring and mesh networks is given in [1]–[3]. The work on routing and grooming optimization for optical networks differentiates with regard to the objectives: throughput maximization for existing networks or network design with minimization of physical resources or CAPEX for given topologies and traffic matrices. In [4], routing and grooming optimization using ILPs and heuristics for WDM mesh networks is presented to maximize the network throughput. The authors of [5] minimize the number of used wavelengths for a network by ILPs and heuristic algorithms. The heuristics yield good solutions in reasonable time. In [6] an overview of optical and electrical

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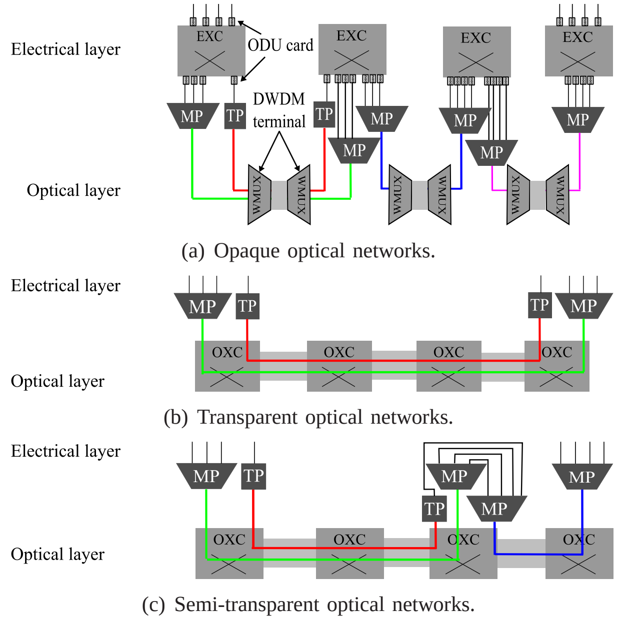

(a) Opaque optical networks.
(b) Transparent optical networks.
(c) Semi-transparent optical networks.

Fig. 1. Connection structures on the optical and electrical layer.

traffic grooming techniques using IP as a client layer is given and a new optical technology for efficient wavelength sharing is introduced and evaluated.

CAPEX minimization requires realistic cost models for optical equipment which have been studied in [7]–[9]. In [10] an ILP for the CAPEX minimization of semitransparent networks is given based on sets of pre-calculated paths and we reuse some of its ideas in our work. A fast heuristic for that formulation is introduced in [11]. Our contribution is a comprehensive description of the CAPEX minimization problem for the three basic DWDM network architectures including 1+1 protection and modular EXC or OADM/OXC costs.

III. Modeling

This section explains the architecture of opaque, transparent, and semi-transparent optical networks from a CAPEX point of view using the cost model presented in [9], [12]. They are all connection-oriented and transmit *optical data unit* (ODU) data streams with bit rates of 2.5, 10, or 40 Gbit/s (ODU1, ODU2, ODU3).

A. Opaque Optical Networks

Opaque optical networks consist of *electrical cross connects* (EXCs) that are connected with their neighbors over fiber lines. EXCs set up ODU connections and switch them in the electrical domain starting at a source node s along a chain of neighboring EXCs and terminating at a destination node d (cf. Fig. 1(a)). Electrical signals are converted for transmission to neighboring nodes into optical signals and are reconverted by the neighbors to electrical signals before they are switched in the electrical domain. An EXC exchanges data streams via tributary ODU interfaces with the upper layer and via trunk ODU interfaces with the optical layer. Hence, a data stream traverses in the network a tributary and a trunk interface at its source node, two trunk interfaces at each intermediate node, and a trunk and tributary interface at its destination node. The cost of these interfaces is denoted by C_b^{odu} and depends on the bit rate b. An EXC has a modular structure. A base node costs $C_{\text{base}}^{\text{exc}}$ and is able to switch 640 Gbit/s. Upgrade units extend the base node by additional 640 Gbit/s and cost $C_{\text{upgrade}}^{\text{exc}}$. An EXC uses a DWDM terminal to enable optical transmission of a fixed number of wavelengths W per fiber link. Transponders (TP) or muxponders (MP) are needed to transmit and receive data over a wavelength. A transponder converts electrical signals from one ODU connection to optical signals that are sent onto one wavelength. A muxponder multiplexes electrical signals from up to 4 ODU connections onto one wavelength and demultiplexes them accordingly. The cost of a DWDM terminal is given by C^{DWDM} and transponder and muxponder costs are denoted by C_b^{tp} and C_b^{mp} where b is their bit rate on the optical layer. Usually, the costs of one muxponder exceeds the costs of four transponders of the next lower linerate. As optical signals attenuate, *optical light amplifiers* (OLAs) are applied about every 80 km along a fiber which leads to C^{ola} costs per km. They just amplify the optical signal. In addition, 3R regenerators are needed approximately every 750 km to cope with fiber lengths of thousands of kilometers. However, we do not take such 3R regenerators into account in our study. Muxponders have several benefits. First, the costs of a single muxponder is often cheaper than the corresponding number of transponders with the same overall bit rate. Second, muxponders reduce the required number of wavelengths and possibly also the required number of fibers which saves the amplifier costs for these fibers and DWDM terminals.

B. Transparent Optical Networks

Transparent optical networks consist of *optical add-drop multiplexers (OADM)* or *optical cross connects* (OXCs) that are connected with their neighbors over a fiber. This costly switching equipment is required only for nodes supporting more than 1 fiber. An OADM with low costs C^{oadm} is sufficient to support up to 2 fibers. An OXC supports 3 to 5 fibers [12, Sect. 3.4.5] and implies base costs $C_{\text{base}}^{\text{oxc}}$ and additional $C_{\text{fbr}}^{\text{oxc}}$ per supported fiber. We do not distinguish any further between OADMS and OXCs. OXCs switch optical signals from incoming fibers to outgoing fibers using the same wavelength. Between two optical switches so-called *optical channels* (OCh) or *lightpaths* are set up. They are connections in the optical domain starting at a source node s along a chain of neighboring OXCs and terminating at a destination node t (cf. Fig. 1(b)). The optical signals are transmitted transparently as they are not converted into the electrical domain at intermediate nodes. Demand streams are directly connected to transponders or muxponders which send their optical signals to DWDM multiplexers already integrated in OXCs. Up to W wavelengths can be multiplexed onto a fiber. OLAs cause the same fiber costs as in Sect. III-A. In contrast to opaque networks, optical signals are not refreshed by *optical-electrical-optical* (OEO) conversion at intermediate hops. Therefore, the limited range of the transponder and muxponder signals of $L^{\max}$ must be respected: lightpaths cannot be longer than this distance. As a lightpath requires the same wavelength on any link within its path, the assignment of wavelengths to lightpaths is not trivial, but we do not consider it in our study.

C. Semi-Transparent Optical Networks

Semi-transparent optical networks use the same hardware components as transparent optical networks. However, OXCs can connect consecutive lightpaths by feeding ODU signals from transponders or muxponders into other transponders or muxponders (cf. Fig. 1(c)). Thus, data are transmitted over a chain of lightpaths. Optical signals are transparently transmitted within lightpaths but not anymore from source to destination of a lightpath chain as OEO-conversion is performed at some intermediate nodes. The physical limit for the length of lightpaths of $L^{\max}$ km still applies, but the length of lightpath chains can go well beyond that limit. Another advantage of lightpath chains is that wavelength conversion can be done where lightpaths are concatenated, i.e., a single wavelength from source to destination is not required. This possibly increases the success probability to establish new connections in a network with already existing connections.

IV. ILPs for CAPEX-Aware Network Design

This section gives a high-level overview of CAPEX-aware network design using *integer linear programs* (ILPs). Then, some terminology and notational conventions are introduced before mathematical descriptions for the network design problems for opaque, transparent, and semi-transparent optical networks are presented in the form of ILPs.

A. Overview

The objective of this paper is the design of optical networks with least installation costs for a given fiber topology and demand matrix with different bit rates. For all considered network types, the routing of the demands and a plan for cost-effective multiplexing using muxponders need to be found so that the equipment required for this path layout has least costs. This is required for networks with and without resilience requirements. In the latter case, each demand is carried over two disjoint paths (1+1 protection) so that one of them still works in case of a fiber cut.

The CAPEX minimization problems are formulated by ILPs. ILPs use integer-valued variables to describe a cost function that is to be minimized while meeting additional constraints for the variables. ILP solvers are programs that find a solution for the variables that minimizes the cost function. ILPs are often very complex and exact solutions cannot be found within manageable time for real-world problem instances. Then, heuristic algorithms may be applied.

B. Terminology and Notational Conventions

To facilitate the readability of our formulae, sets are denoted by calligraphic letters, parameters and constants by uppercase letters, while variables by lowercase letters.

The fiber topology is given by a graph $G_F = (\mathcal{V}, \mathcal{E}_F)$ where $\mathcal{V}$ is the set of nodes and $\mathcal{E}_F \subseteq \mathcal{V} \times \mathcal{V}$ is the set of fiber links connecting them. G_F is also called the *fiber layer*. The length of a fiber link $(i,j) \in \mathcal{E}_F$ is given by L_{ij} and the number of activated fibers on that link is described by the variable $f_{ij} \in \mathbb{N}$. Each fiber can carry at most W wavelengths.

The *demand layer* is defined by $G_D = (\mathcal{V}, \mathcal{E}_D)$ with $\mathcal{E}_D = \mathcal{V} \times \mathcal{V}$. The demand matrix is three-dimensional and contains the number D^b_{sd} of demands for each bit rate $b \in \mathcal{B} = \{2.5, 10, 40\}$ Gbit/s and for each source-destination pair $(s,d) \in \mathcal{E}_D$. Demands of $(s,d) \in \mathcal{E}_D$ with bit rate b are numbered $0 \leq k < D^b_{sd}$ and are identified by the quadruple $bsdk$.

In opaque networks, a demand $bsdk$ is routed via a primary path p^{bsdk} and possibly also over a backup path q^{bsdk} through the fiber topology. The binary variables $p^{bsdk}_{ij}, q^{bsdk}_{ij} \in \{0,1\}$ indicate whether these paths contain link $(i,j) \in \mathcal{E}_F$, i.e., they describe the path layout. In transparent networks, potential lightpaths sdk instead of demands $bsdk$ are routed per source-destination pair $(s,d) \in \mathcal{E}_D$ and numbered by $0 \leq k < K$ where K is just an upper bound on the number of required lightpaths. Their paths are given by binary variables $p^{sdk}_{ij}, q^{sdk}_{ij}$ analogously to paths of demands. Non-empty paths are provided only for a usually smaller number of required lightpaths and the number of potential lightpaths K is further qualified in Sect. IV-D and Sect. IV-E by D^{sum}_{sd} and D^{sum}. This cumbersome construction is needed for the sake of a linear program formulation. Semi-transparent networks require an additional *lightpath layer* $G_L = (\mathcal{V}, \mathcal{E}_L)$ where $\mathcal{E}_L \subseteq \mathcal{V} \times \mathcal{V}$ indicates potential lightpath connectivity, i.e., it is not clear whether a lightpath will be set up between two nodes $(x,y) \in \mathcal{E}_L$. In contrast to transparent networks, lightpaths xyk are established for $(x,y) \in \mathcal{E}_L$ instead for $(s,d) \in \mathcal{E}_D$, but all other lightpaths issues are the same.

The use of $\mathcal{E}_F, \mathcal{E}_L, \mathcal{E}_D$ instead of $\mathcal{V} \times \mathcal{V}$ in formulae provides layer information which improves their comprehensiveness. The number of transponder- and muxponder-based lightpaths with an optical transmission rate $b \in \mathcal{B}$ are given by the variables t^b and m^b: $t_{ij}, m_{ij}, (i,j) \in \mathcal{E}_F$ describe their number per link in opaque networks, $t_{sd}, m_{sd}, (s,d) \in \mathcal{E}_D$ describe their number per source-destination pair in transparent networks, and $t_{xy}, m_{xy}, (x,y) \in \mathcal{E}_L$ describe their number per lightpath connectivity in semi-transparent networks. Component costs are denoted by the parameters C^X or C^X_Y and explained when needed.

C. Opaque Optical Networks

We first present the CAPEX value for survivable and non-survivable opaque networks as the objective function for the CAPEX minimization problem. Then we add routing constraints for flow conservation and disjoint primary and backup paths and provide lower bounds for the required hardware equipment. Finally, we recapitulate the ILP structure.

1) CAPEX: The overall CAPEX for non-survivable opaque networks are summarized by Eqn. (1). They consist of the costs for two ODU cards on EXC tributary interfaces for each demand, two ODU cards on EXC trunk interfaces for each demand and each traversed link $(i,j)\in\mathcal{E}_F$, two transponders and muxponders for all transponder- and muxponder-based optical transmissions per link (t_{ij}^b, m_{ij}^b), two DWDM multiplex terminals for all activated fibers f_{ij} on all links, the OLAs which are proportional to the length of activated fibers, one EXC base node per node $v\in\mathcal{V}$ and the EXC upgrade units u_v per node $v\in\mathcal{V}$.

$$c_1^{\text{opaque}} = \sum_{(s,d)\in\mathcal{E}_D, b\in\mathcal{B}} 2\cdot C_b^{\text{odu}}\cdot D_{sd}^b + \sum_{\substack{(s,d)\in\mathcal{E}_D, b\in\mathcal{B},\\ (i,j)\in\mathcal{E}_F, 0\le k<D_{sd}^b}} 2\cdot C_b^{\text{odu}}\cdot p_{ij}^{bsdk} + \sum_{(i,j)\in\mathcal{E}_F, b\in\mathcal{B}} \left(2\cdot C_b^{\text{tp}}\cdot t_{ij}^b + 2\cdot C_b^{\text{mp}}\cdot m_{ij}^b\right) + \sum_{(i,j)\in\mathcal{E}_F}\left(2\cdot C^{\text{DWDM}} + C^{\text{ola}}\cdot L_{ij}\right)\cdot f_{ij} + \sum_{v\in\mathcal{V}}\left(C_{\text{base}}^{\text{exc}} + C_{\text{upgrade}}^{\text{exc}}\cdot u_v\right). \quad (1)$$

1+1 dedicated path protection sets up a primary path p^{bsdk} and a link-disjoint backup path q^{bsdk} for each demand $bsdk$. This causes additional costs for ODU cards, transponders, muxponders, fibers, and EXC upgrade units. All of them are covered in Eqn. (1) by the component costs except for the ODU cards. Thus, we add for the backup paths two ODU cards on EXC trunk interfaces per demand and traversed link assuming that EXC equipment doubles signals received from tributary interfaces onto primary and backup paths and merges them at the destination. Thus, the CAPEX for survivable opaque networks are

$$c_{1+1}^{\text{opaque}} = c_1^{\text{opaque}} + \sum_{\substack{(s,d)\in\mathcal{E}_D, b\in\mathcal{B},\\ (i,j)\in\mathcal{E}_F, 1\le k\le D_{sd}^b}} 2\cdot C_b^{\text{odu}}\cdot q_{ij}^{bsdk}. \quad (2)$$

2) Routing Constraints for Individual Demands: Flow conservation means that the path p^{bsdk} of a demand $bsdk$ leaves only the source node s, enters only the destination node d, and both enter and leave any intermediate nodes. These constraints are captured by the following formulae:

$$\forall b\in\mathcal{B}, (s,d)\in\mathcal{E}_D, 0\le k<D_{sd}^b, \forall i\in\mathcal{V}: \quad \sum_{(i,j)\in\mathcal{E}_F} p_{ij}^{bsdk} - \sum_{(j,i)\in\mathcal{E}_F} p_{ji}^{bsdk} = \begin{cases} 1 & i=s \\ -1 & i=d \\ 0 & \text{otherwise.}\end{cases} \quad (3)$$

When resilience is needed, a demand $bsdk$ requires a backup path q^{bsdk} which is also subject to flow conservation analogously to Eqn. (3). In addition, primary and backup paths must be link-disjoint. The links $(i,j),(j,i)\in\mathcal{E}_F$ use the same physical resources. Therefore, at most one of them can be used either by the primary or backup path. This is asserted by

$$\forall b\in\mathcal{B}, (s,d)\in\mathcal{E}_D, 0\le k<D_{sd}^b, (i,j)\in\mathcal{E}_F: \quad p_{ij}^{bsdk} + p_{ji}^{bsdk} + q_{ij}^{bsdk} + q_{ji}^{bsdk} \le 1. \quad (4)$$

3) Lower Bounds for Required Hardware Components: The cost functions in Eqns. (1) and (2) require the number of transponders t_{ij}^b, muxponders m_{ij}^b, and fibers f_{ij} per link $(i,j)\in\mathcal{E}_F$ and the number of EXC upgrade units u_v per node $v\in\mathcal{V}$. The minimization of the cost functions implies also the minimization of the number of these hardware components. To assure that the network can carry the demand matrix with the desired protection, we derive conditions providing a lower bound for their number. The conditions are formulated for primary and backup paths. When survivability is not needed, the terms for the backup path are removed, but we omit these slightly simplified formulae for the sake of brevity.

First, we provide lower bounds on the number of transponders and muxponders. Each transponder serves up to one demand of the same bit rate while each multiplexer serves up to four demands with one fourth of its bit rate. Thus, the number of transponders and muxponders on any link is constraint by the number of demands that are carried over them:

$$\forall (i,j)\in\mathcal{E}_F: \; t_{ij}^{2.5} + 4\cdot m_{ij}^{10} \ge \sum_{(s,d)\in\mathcal{E}_D, 0\le k<D_{sd}^{2.5}} \left(p_{ij}^{2.5sdk} + q_{ij}^{2.5sdk}\right) \quad (5a)$$

$$\forall (i,j)\in\mathcal{E}_F: \; t_{ij}^{10} + 4\cdot m_{ij}^{40} \ge \sum_{(s,d)\in\mathcal{E}_D, 0\le k<D_{sd}^{10}} \left(p_{ij}^{10sdk} + q_{ij}^{10sdk}\right) \quad (5b)$$

$$\forall (i,j)\in\mathcal{E}_F: \; t_{ij}^{40} = \sum_{(s,d)\in\mathcal{E}_D, 0\le k<D_{sd}^{40}} \left(p_{ij}^{40sdk} + q_{ij}^{40sdk}\right) \quad (5c)$$

Wavelengths are not directed and can be operated in any direction. A lower bound for them is given by the number of transponder- and muxponder-based lightpaths traversing them in any direction and the maximum number of wavelengths W per fiber. In order to count wavelengths only once for both directions, $i<j$ has to be ensured:

$$\forall (i,j)\in\mathcal{E}_F, i<j: f_{ij}\cdot W \ge \sum_{b\in\mathcal{B}}\left(t_{ij}^b + t_{ji}^b + m_{ij}^b + m_{ji}^b\right). \quad (6)$$

Each EXC consists of a base node with a transmission capacity of $R_{\text{base}} = 640$ Gbit/s and possibly several upgrade units with $R_{\text{upgrade}} = 640$ Gbit/s each. Their minimum number $u_v\in\{0,1,2,4\}$ for a specific node $v\in\mathcal{V}$ is a variable and bounded by the traffic rate switched by the EXC:

$$\forall v\in\mathcal{V}: \quad R_{\text{base}} + u_v\cdot R_{\text{upgrade}} \ge \sum_{\substack{(s,d)\in\mathcal{E}_D,\\ (i,j)\in\mathcal{E}_F: j=v\vee(s=v\wedge i=v)}} \sum_{b\in\mathcal{B}} b\cdot \sum_{0\le k<D_{sd}^b}\left(p_{ij}^{bsdk} + q_{ij}^{bsdk}\right) \quad (7)$$

4) Summary of the ILPs: The ILP for the CAPEX minimization of survivable opaque networks minimizes the objective function in Eqn. (2). The free variables $t_{ij}^b, m_{ij}^b, f_{ij}, p_{ij}^{bsdk}, q_{ij}^{bsdk}, u_v$ with $b\in\mathcal{B}, (s,d)\in\mathcal{E}_D, (i,j)\in\mathcal{E}_F, 0\le k<D_{sd}^b, v\in\mathcal{V}$ are subject to the constraints in Eqns. (3)–(7). A modified version of Eqn. (3) also applies to backup paths q^{bsdk}. The ILP for the CAPEX minimization of non-survivable opaque networks minimizes the objective function in Eqn. (1). It has the same free variables except for the backup paths q^{bsdk} and respects the same constraints except for Eqn. (4) which guarantees link-disjointness for primary and backup paths.

D. Transparent Optical Networks

We describe the CAPEX-minimization problem for transparent networks analogously to Sect. IV-C.

1) CAPEX: Eqn. (8) sums up the CAPEX for non-survivable transparent networks. They consist of the costs for two transponders or muxponders per transponder- or muxponder-based lightpath for any source-destination pair, the fiber costs induced by the OLAs, and the OADM/OXC nodes. The binary variable a_v indicates whether node v supports at least two fibers and requires at least a OADM. The binary variable o_v indicates whether node v supports at least three fibers and requires an OXC instead of a OADM. If more than two fibers are needed at node v, the additional fibers are calculated by the variable e_v.

$$\begin{aligned} c_1^{\text{trans}} \;=\; & \sum_{(s,d)\in\mathcal{E}_D, b\in\mathcal{B}} \left(2\cdot C_b^{\text{tp}}\cdot t_{sd}^b + 2\cdot C_b^{\text{mp}}\cdot m_{sd}^b\right) + \\ & \sum_{(i,j)\in\mathcal{E}_F} C^{\text{ola}}\cdot L_{ij}\cdot f_{ij} + \sum_{v\in\mathcal{V}} (C^{\text{oadm}}\cdot a_v + \\ & \left(-C^{\text{oadm}} + C_{\text{base}}^{\text{oxc}} + 2\cdot C_{\text{fbr}}^{\text{oxc}}\right)\cdot o_v + C_{\text{fbr}}^{\text{oxc}}\cdot e_v). \end{aligned} \quad (8)$$

1+1 dedicated path protection sets up a primary path and a link-disjoint backup path for each demand. This causes additional costs for transponders, muxponders, and fibers. This needs to be taken into account for the cost function of survivable transparent networks:

$$c_{1+1}^{\text{trans}} \;=\; c_1^{\text{trans}} + \sum_{(s,d)\in\mathcal{E}_D, b\in\mathcal{B}} \left(2\cdot C_b^{\text{tp}}\cdot t_{sd}^b + 2\cdot C_b^{\text{mp}}\cdot m_{sd}^b\right) \quad (9)$$

The increased number of fibers is already covered by f_{ij} and taken into account by c_1^{trans} in Eqn. (8).

2) Lower Bound for the Number of Lightpaths: In transparent networks, direct lightpaths are set up to carry demands from source to destination. Several demands are possibly multiplexed onto a single lightpath using muxponders. Similarly to Eqn. (5), a lower bound for the number of transponder- and muxponder-based lightpaths is given by

$$\forall (s,d)\in\mathcal{E}_D: \quad t_{sd}^{2.5} + 4\cdot m_{sd}^{10} \geq D_{sd}^{2.5} \quad (10a)$$
$$\forall (s,d)\in\mathcal{E}_D: \quad t_{sd}^{10} + 4\cdot m_{sd}^{40} \geq D_{sd}^{10} \quad (10b)$$
$$\forall (s,d)\in\mathcal{E}_D: \quad t_{sd}^{40} = D_{sd}^{40}. \quad (10c)$$

3) Routing Constraints for Individual Lightpaths: An upper bound for the number of potential lightpaths for a source-destination pair $(s,d)\in\mathcal{E}_D$ is the sum of all its demands $D_{sd}^{sum} = \sum_{b\in\mathcal{B}} D_{sd}^b$. Each of them has a path p_{ij}^{sdk}, $0\leq k < D_{sd}^{sum}$, but the number of required lightpaths is determined by the sum of all transponder- and muxponder-based lightpaths for (s,d). As the path variables p_{ij}^{sdk} are binary, this can be described by

$$\sum_{\substack{(s,d)\in\mathcal{E}_D, (s,i)\in\mathcal{E}_F \\ 0\leq k< D_{sd}^{sum}}} p_{si}^{sdk} = t_{sd}^{2.5} + t_{sd}^{10} + t_{sd}^{40} + m_{sd}^{10} + m_{sd}^{40}. \quad (11)$$

When the lightpaths are routed over multiple hops through the fiber topology, their path layout must respect constraints for flow conservation, length restrictions, and link-disjointness for primary and backup paths when resilience is required. The flow conservation rules for lightpaths are essentially the same as for the demands in Eqn. (12) and must be fulfilled for all D_{sd}^{sum} potential lightpaths:

$$\forall (s,d)\in\mathcal{E}_D, 0\leq k< D_{sd}^{sum}, \forall i\in\mathcal{V}: \quad \sum_{(i,j)\in\mathcal{E}_F} p_{ij}^{sdk} - \sum_{(j,i)\in\mathcal{E}_F} p_{ji}^{sdk} = \begin{cases} 1 & i=s \\ -1 & i=d \\ 0 & \text{otherwise.}\end{cases} \quad (12)$$

As optical signals on transparent lightpaths cannot be electrically refreshed, paths cannot be longer than $L^{\max}$ km. Thus, the following length restriction applies to the layout of all lightpaths:

$$\forall (s,d)\in\mathcal{E}_D, 0\leq k< D_{sd}^{sum}: \sum_{(i,j)\in\mathcal{E}_F} L_{ij}\cdot p_{ij}^{sdk} \leq L^{\max} \quad (13)$$

When resilience is needed, a lightpath sdk requires in addition to its primary path p^{sdk} a backup path q^{sdk} which is also subject to the number of required lightpaths, flow conservation, and length restriction analogously to Eqns. (11)–(13). In addition, primary and backup paths must be link-disjoint which is asserted similarly to Eqn. (4) by

$$\begin{aligned} & \forall (s,d)\in\mathcal{E}_D, 0\leq k< D_{sd}^{sum}, (i,j)\in\mathcal{E}_F: \\ & p_{ij}^{sdk} + p_{ji}^{sdk} + q_{ij}^{sdk} + q_{ji}^{sdk} \leq 1. \end{aligned} \quad (14)$$

4) Lower Bound for the Number of Fibers: Again we assume without loss of generality that the number of fibers f_{ij} is positive only for $i<j$. A lower bound for them is given by the number of transponder- and muxponder-based lightpaths traversing them in any direction and the maximum number of wavelength W per fiber:

$$\begin{aligned} & \forall (i,j)\in\mathcal{E}_F, (i<j): \\ & f_{ij}\cdot W \geq \sum_{(s,d)\in\mathcal{E}_D, 0\leq k<D_{sd}^{sum}} \left(p_{ij}^{sdk} + p_{ji}^{sdk} + q_{ij}^{sdk} + q_{ji}^{sdk}\right) \end{aligned} \quad (15)$$

5) Lower Bounds for Switching Equipment: The binary variables a_v and o_v are one if the number of activated fibers attached to a node is larger than 1 or 2 and zero otherwise. They indicate if at least an OADM or even an OXC is required. The integer variable $e_v\in\{0,1,2,3\}$ indicates for the OXC at v the minimum number of required fiber cards above 2; e_v is at most 3 as an OXC can support at most 5 fibers.

$$\forall v\in\mathcal{V}: \quad 4\cdot a_v + 1 \;\geq \sum_{(i,v),(v,i)\in\mathcal{E}_F} f_{iv} \quad (16a)$$
$$\forall v\in\mathcal{V}: \quad 3\cdot o_v + 2 \;\geq \sum_{(i,v),(v,i)\in\mathcal{E}_F} f_{iv} \quad (16b)$$
$$\forall v\in\mathcal{V}: \quad e_v + 2 \;\geq \sum_{(i,v),(v,i)\in\mathcal{E}_F} f_{iv} \quad (16c)$$

6) Summary of the ILPs: The ILP for the CAPEX minimization of survivable transparent networks minimizes the objective function in Eqn. (9). The free variables $t_{sd}^b, m_{sd}^b, f_{ij}, p_{ij}^{sdk}, q_{ij}^{sdk}$ with $b\in\mathcal{B}, (s,d)\in\mathcal{E}_D, (i,j)\in\mathcal{E}_F, 0\leq k< D_{sd}^{sum}$ are subject to the constraints in Eqns. (11)–(16). Modified versions of Eqns. (11)–(13) also apply to backup lightpaths q^{sdk}. The ILP for the CAPEX minimization of non-survivable transparent networks minimizes the objective function in Eqn. (8). It has the same free variables except for the backup lightpaths q^{sdk} and respects the same constraints except for Eqn. (14) which guarantees link-disjointness for primary and backup lightpaths.

E. Semi-Transparent Optical Networks

We describe the CAPEX-minimization problem for semi-transparent networks analogously to Sections IV-C and IV-D.

1) CAPEX: Semi-transparent and transparent networks consist of the same hardware components. Therefore, the CAPEX of semi-transparent networks with and without resilience requirements can also be calculated by Eqn. (8).

2) Routing Constraints on the Lightpath Layer: In semi-transparent networks, demands are routed over lightpath chains instead of a single lightpath as in transparent networks. Survivability can be achieved on the fiber or the lightpath layer: either primary and backup path are provided for each lightpath or a primary and backup lightpath chain is provided for each demand. In this work we follow the first approach. The variable g_{xy}^{bsd} indicates how many demands with bit rate b are routed over a lightpath from x to y, i.e. $(x,y) \in \mathcal{E}_L$. Flow conservation also applies to lightpath chains on the lightpath layer and is similar to those on the fiber layer (cf. Eqns. (3) and (12)). However, flow conservation for aggregated demands suffices in this case as protection is not provided on the lightpath layer:

$$\forall (s,d) \in \mathcal{E}_D, b \in \mathcal{B}, \forall x \in \mathcal{V}: \sum_{(x,y)\in\mathcal{E}_L} g_{xy}^{bsd} - \sum_{(y,x)\in\mathcal{E}_L} g_{yx}^{bsd} = \begin{cases} D_{sd}^b & x=s \\ -D_{sd}^b & x=d \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

3) Lower Bound for the Number of Lightpaths: The number of required lightpaths is determined by the number g_{xy}^{bsd} of demands routed over them and potential multiplexing of these demands by muxponders. Thus, the number of transponder- and multiplexer-based lightpaths with different bit rates is constraint by

$$\forall (x,y) \in \mathcal{E}_L: \quad t_{xy}^{2.5} + 4 \cdot m_{xy}^{10} \geq \sum_{(s,d)\in\mathcal{E}_D} g_{xy}^{2.5sd} \quad (18a)$$

$$\forall (x,y) \in \mathcal{E}_L: \quad t_{xy}^{10} + 4 \cdot m_{xy}^{40} \geq \sum_{(s,d)\in\mathcal{E}_D} g_{xy}^{10sd} \quad (18b)$$

$$\forall (x,y) \in \mathcal{E}_L: \quad t_{xy}^{40} = \sum_{(s,d)\in\mathcal{E}_D} g_{xy}^{40sd}. \quad (18c)$$

4) Lower Bound for the Number of Fibers: The number of potential lightpaths per lightpath connectivity $(x,y) \in \mathcal{E}_L$ is bounded only by the number of all demands in the network $D^{sum} = \sum_{(s,d)\in\mathcal{E}_D} \sum_{b\in\mathcal{B}} D_{sd}^b$. However, lightpaths are only required for all transponder- and muxponder-based lightpaths for this lightpath connectivity (x,y) and an equation similar to Eqn. (11) applies. The lightpaths are routed on the fiber layer respecting the constraints for their maximum number, flow conservation, link disjointness, and maximum length as in Sect. IV-D3. Then, a lower bound for the number of fibers can be obtained by Eqn. (15).

5) Summary of the ILPs: The ILP for the CAPEX minimization of survivable opaque networks minimizes the objective function in Eqn. (9). The free variables $t_{xy}^b, m_{xy}^b, f_{ij}, p_{ij}^{xyk}, q_{ij}^{xyk}, g_{xy}^{bsd}$ with $b \in \mathcal{B}, (s,d) \in \mathcal{E}_D, (x,y) \in \mathcal{E}_L, (i,j) \in \mathcal{E}_F, 0 \leq k < D^{sum}$ are subject to the constraints in Eqns. (17) and (18). Furthermore, the constraints in Eqns. (11)–(16) and modified versions of Eqns. (11)–(13) apply for the backup lightpaths. Note that the minimization problem for semi-transparent networks is more complex than for transparent networks due to the additional variables g_{xy}^{bsd} and due to the larger number of potential lightpaths p^{xyk}, q^{xyk} which are D^{sum} for transparent networks and $D^{sum} \cdot |\mathcal{V}| \cdot (|\mathcal{V}|-1)$ for semi-transparent networks. The ILP for the CAPEX minimization of non-survivable semi-transparent networks minimizes the objective function in Eqn. (8). It has the same free variables except for the backup lightpaths q^{xyk} and respects the same constraints except for Eqn. (14) which guarantees link-disjointness for primary and backup lightpaths.

V. Conclusion

We have reviewed the basic architecture and component costs of opaque, transparent, and semi-transparent networks based on the new cost models gained from the Nobel-2 project [7]. We modeled for them the network design problem from a CAPEX point of view using integer linear programs (ILPs). The output of the ILPs is a least-cost network installation plan including hardware equipment, routing, and multiplexing information for a given fiber topology that satisfies a demand matrix with different bit rates. When resilience is needed, primary and link-disjoint backup paths are provided for 1+1 protection against single-fiber cuts. The value of this work is the canonical presentation of the optimization problems for the three network types. The ILPs are easily extensible, give insights into the structure of the optimization problems, and make their differences obvious. Their complexity is high so that real-world problem instances cannot be solved efficiently by ILP solvers. However, they provide clear problem formulations that may be tackled by heuristics in the future.

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