A Study for Merging of Automated Vehicles

Yildirim Dülgar, Hubert Rehborn, Sven-Eric Molzahn, Micha Koller
Mercedes-Benz R&D, Daimler AG, HPC: 059-X901, D-71059 Sindelfingen, Germany

Michael Menth
Universität Tübingen, Fachbereich Informatik, Lehrstuhl für Kommunikationsnetze, Sand 13, D-72076 Tübingen, Germany

Boris Kerner, Michael Schreckenberg
Universität Duisburg-Essen, Physik von Transport und Verkehr, Lotharstr. 1, D-47057 Duisburg, Germany

Abstract

An automated vehicle should perform all its driving maneuvers safely and consistently with all laws, including merging maneuvers from an on-ramp onto the freeway. In this study an analysis model will be developed to investigate the expected waiting times for automated vehicles during merging maneuvers at freeway on-ramps. Single vehicle data measured at single freeway locations are used. The study reveals that the merging time threshold, which is the time an automated vehicle needs for a complete merging maneuver, is a crucial parameter for automated vehicles. Furthermore, it is shown by using real empirical single vehicle data that the merging possibility of automated vehicles is lane and traffic phase dependent in the scope of Kerner's three-phase traffic theory: On the right lane and in synchronized flow the merging possibilities have lowest values and the waiting time is highest.

1 Introduction

Merging of automated vehicles at freeway on-ramps onto the main freeway is a relevant topic. On a longer route, merging maneuvers might occur quite often. All driving maneuvers, including a merging maneuver, done by an automated vehicle have to be safe and consistent with all laws. Therefore, an automated vehicle has to keep a safety time gap to the following vehicle during a merging maneuver. However, human drivers in some cases do not keep the safety time gap to the following vehicle during a merging maneuver in real traffic. In this study, we will discuss the question if an automated vehicle would be able to complete a merging maneuver safely and by keeping the safety time gap at any time.

In this study an analysis model based on real single vehicle data is developed to investigate the expected waiting times for automated vehicles during a merging maneuver at freeway on-ramps. Time gap information between vehicles measured at single freeway locations are used for this investigation. If time gaps between consecutive vehicles are too small during a time interval, the automated vehicle is not able to merge from the on-ramp onto the main freeway and, therefore, it has to wait. The study reveals that the merging time threshold, which is the time an automated vehicle needs for a complete and safe merging maneuver, is a crucial parameter for automated vehicles.

The study shows that large merging time thresholds of automated vehicles could significantly delay or even hinder an automated vehicle to merge. Therefore, additional sensors which measure time headways between vehicles at the merging area could be helpful to support the merging maneuver of automated vehicles in real traffic environments. If the sensors measure higher headways on the right lane the merging possibility is higher. If additional sensors are used, low latency could be important because the time which an automated vehicle needs for a safe merging maneuver would be not only the merging time threshold itself but also the latency between the time headway measurement and the information reception in the automated vehicle. Due to very low latency, Mobile-Edge-Computing could be potentially a good choice [1].

We analyze measured time gap distributions of human drivers on different lanes and in different traffic phases. The detailed explanation and theory of traffic phases is based on Kerner's three-phase traffic theory [2-7] which is explained in Section 3. By using real empirical single vehicle data this study shows that the merging possibility of automated vehicles is lane and traffic phase dependent. The merging possibility features for the different lanes and traffic phases are shown and discussed in detail.

This paper is organized as follows. In Section 2 the related work for this study is discussed. Some theoretical elements of Kerner's three-phase theory [2-7] are introduced in Section 3. In Section 4 the measurements used in this study are described and a three-phase analysis is presented. Furthermore, in Section 5 we investigate the expected waiting time of an automated vehicle at an on-ramp with regard to the merging time threshold, which an automated vehicle needs to complete a safe merging maneuver from an on-ramp onto the freeway. In Section 6 the essential conclusions are shown.

2 Related Work

In [8] merging maneuvers are analyzed based on empirical data collected by a camera installed on a helicopter. It is found that the merge location depends on the traffic flow which is congested or free flow. The data set, however, only consists of 35 minutes time duration on a 400 m freeway. Therefore, two important aspects could not be considered regarding time gaps: statistical properties and the dependency on traffic over the time of day.

There are two different lane changes: mandatory lane changes (MLC), e.g., merging from an on-ramp onto the freeway, and discretionary lane changes (DLC), e.g., changing onto the most left lane to drive faster [9-15]. It is expected that the time duration for mandatory lane changes is shorter than for discretionary lane changes [16]. Moreover, small space gaps for merging are accepted by drivers who are changing the lane mandatory [8,10,17-20].

Cooperative merging by increasing the time gap to the following vehicle was investigated in [21-23] and validated by [17] using NGSIM data [24]. Characteristics of different driving maneuvers, including merging maneuvers, and the effect of freeway infrastructure and traffic flow on these characteristics are discussed in [17].

A merging maneuver of an automated vehicle has to be safe and consistent with all laws. Therefore, an automated vehicle has to keep a safety time gap to the following vehicle during a merging maneuver at any time. However, human drivers in some cases do not keep the safety time gap to the following vehicle during a merging maneuver in real traffic. In [25] a time gap distribution measured by 24 human drivers over a time interval of nine hours is given. It shows that more than 80% of the time gaps of the human drivers are below the safety time gap. Thus, a vehicle which is keeping the safety time gap at any time, like an automated vehicle, could have problems in real traffic, e.g., in case of a merging maneuver.

In addition to the time headway distributions discussed in Section 4.1, we will investigate the waiting time of an automated vehicle which is trying to merge from an on-ramp onto the freeway in Section 4.2. The automated vehicle is only able to merge onto the freeway if there is a time gap between consecutive vehicles that is equal or greater than the merging time threshold that the automated vehicle needs for a safe merging maneuver. Otherwise, it has to wait. We note that the needed time gap for merging describes an optimal case for which the automated vehicle can use this time gap. In many real cases other vehicles in flowing traffic might hinder the automated vehicles to merge. We get the time gap data between consecutive vehicles on the freeway from induction loop detectors which are embedded in each freeway lane. The data used in this study is described in Section 4 in detail. A crucial value for a merging maneuver is the merging time which is the duration an automated vehicle needs to change a lane. However, not only the merging time is needed for a safe merging maneuver, but also a safety time gap to the following vehicle and the vehicle behind. We call the merging time with the safety time gap as merging time threshold. We note that the merging time threshold depends on several factors, e.g., traffic, road condition, road gradient, weather condition. For the automated vehicles we estimate 4 s for a comfortable and safe merging maneuver, i.e., the merging time threshold is 4 s. We will also use other values for analysis purposes, e.g., 2 s, 5 s and 6 s. In [26] a detailed analysis of the lane change duration is done by using NGSIM data, [24], which is a very detailed data set of real human drivers on a freeway. The results in [26] show for the data [24] that the most common lane change duration of real drivers is about 3 s and the mean and standard variation of the lane change duration distribution is 4.01 ± 2.31 s. However, if we take the preparation of a merging maneuver into account, the lane change duration of a human driver might take about 5 - 6 s. This duration is for the whole merging maneuver independent of the time gap distribution as in [25].

3 Theoretical Background: Elements of Three-Phase Traffic Theory

To put the time headway investigation in the context of three-phase traffic theory, we will briefly mention some fundamentals of this theory. Traffic can be divided into two categories: free flow and congested traffic. Based on spatiotemporal analysis of empirical freeway traffic data, Kerner found that congested traffic, in turn, has two different phases: synchronized flow and wide moving jam [2-7]. Hence, there are three traffic phases: free flow, synchronized flow and wide moving jam. To differentiate between the two congested traffic phases, synchronized flow and wide moving jam, the following macroscopic criterion has to be considered. A wide moving jam is propagating upstream and is spatially restricted by a downstream and upstream jam front. The main feature of wide moving jam is that when it propagates upstream, it maintains the mean velocity of the downstream jam front without being disturbed by freeway bottlenecks or other traffic phases. In contrast, synchronized flow does not exhibit this feature. In empirical traffic data it can be observed that the downstream front of synchronized flow is often fixed at a freeway bottleneck.

In the study, detectors embedded in each lane measure time gaps between consecutive vehicles at some freeway locations. Since we aim at investigating the time gap data in the different traffic phases, free flow, synchronized flow and wide moving jam, we need to distinguish between them based on local detector measurements. Time intervals for the time gap data with speeds above 80 km/h can be considered as free flow. It is more difficult to distinguish between synchronized flow and wide moving jam. In wide moving jam traffic flow interruptions can be observed, which occur due to vehicles stopping or driving very slowly within the jam, i.e., the local detector should measure very large time gaps between at least two consecutive vehicles. The condition for wide moving jam is the following microscopic criterion [3-5]

$$\tau_{max} \gg \tau_{del}^{(ac)}, \tag{1}$$

where τ_{max} is the maximum time gap between two consecutive vehicles within the jam and $\tau_{del}^{(ac)}$ is the mean time delay in vehicle acceleration at the downstream jam front from a standstill within the jam. According to empirical results $\tau_{del}^{(ac)} \approx 1.5 - 2\ s$ [3,4,7]. If Equation (1) is satisfied, there are vehicles in the jam which are in a standstill or moving at very low speed compared to the jam inflow and outflow speed.

4 Traffic Measurement and Three-Phase Analysis

The data used in this study is measured on different days in December 1995 and June 1996 at different locations at the three-lane German freeway A5 in Hessen, see Fig. 1. A detector set which consists of two double induction loop detectors is embedded in each freeway lane. The detector registers each vehicle which is crossing the detector by electromagnetic induction. Therefore, each crossing vehicle is indicated by an electromagnetic impulse over time as well as the time when no vehicle passes the detector. This allows to calculate the gross time gap $\tau_{i,i+1}^{(gross)}$ between two consecutive vehicles i and $\mathrm{i}+1$. Besides the gross time gap $\tau_{i,i+1}^{(gross)}$, we also get the vehicle speed v_i, the vehicle length d_i and the time duration Δt_i, which takes a vehicle to cross the detector from the beginning to the end of the detector. The net time gap (time headway) between two consecutive vehicles i and $\mathrm{i}+1$ is calculated as follows: $\tau_{i,i+1} = \tau_{i,i+1}^{(gross)} - \Delta t_i$. In this paper, we will use the terms net time gap, time gap, and time headway synonymously.

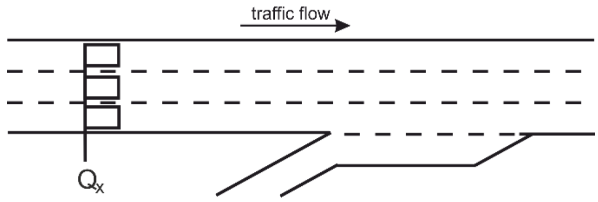


Fig. 1: Roadside measurements of time gaps at a freeway infrastructure with an on-ramp and merging section.

4.1 Statistical Properties

To illustrate the measured data, Fig. 2 shows the measured time headway distribution of the whole data set of approximately 25.000 time headway measurements separated by right and left lane. It is clearly visible that the time headways strongly depend on the related lane with a clear shift for the right lane distribution to higher values. For the left lane the peak of time headways is below 2 s. It should be noted that all time headways are measured at higher flow rates, i.e., all lanes are covered with vehicles.

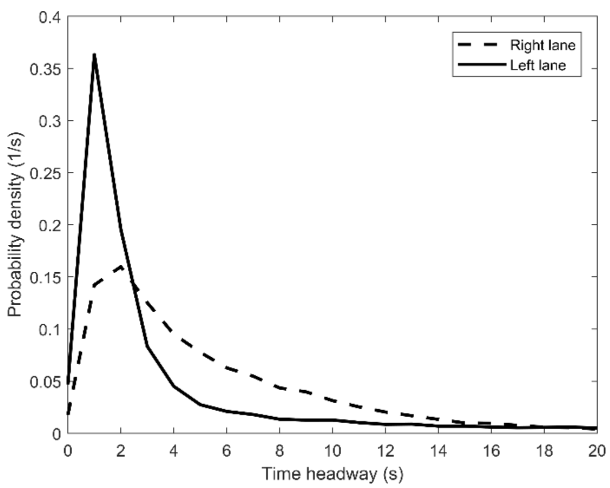


Fig. 2: Time headway distributions of approximately 25.000 single vehicle measurements for the right and left lane of a three-lane highway.

The traffic theory on which this paper is based is the three-phase traffic theory described in Section 3 and in [2-7]. In Fig. 3 each bullet illustrates the speed (top picture) of one vehicle and the related time headway (bottom picture): some very long time headways can be observed in a wide moving jam with time headways above 10 s. This is due to vehicles which are stopping or driving very slowly within the jam. Thus, a "flow-interruption" effect occurs in a wide moving jam whereas it does not occur in synchronized flow.

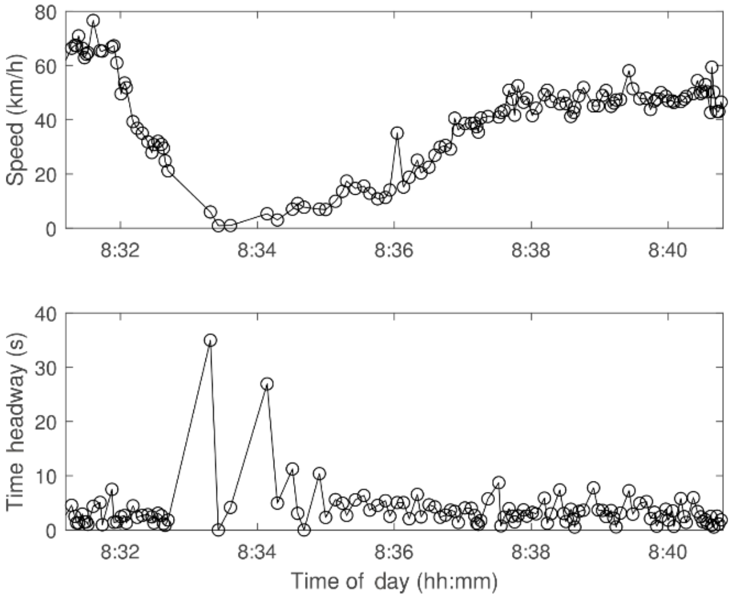


Fig. 3: A "flow-interruption" effect in a wide moving jam. Large time headways inside a wide moving jam are observed due to vehicles which are stopping or driving very slow within the jam [3,4].

4.2 Three-Phase Traffic Analysis

We aim to separate the time headway measurements into traffic phases. In Section 3, we described how to distinguish between all three traffic phases: free flow (F), synchronized flow (S) and wide moving jam (J). The microscopic criteria $\tau_{max} \gg \tau_{del}^{(ac)}$ is used to distinguish between the two congested traffic phases: synchronized flow and wide moving jam [2-7].

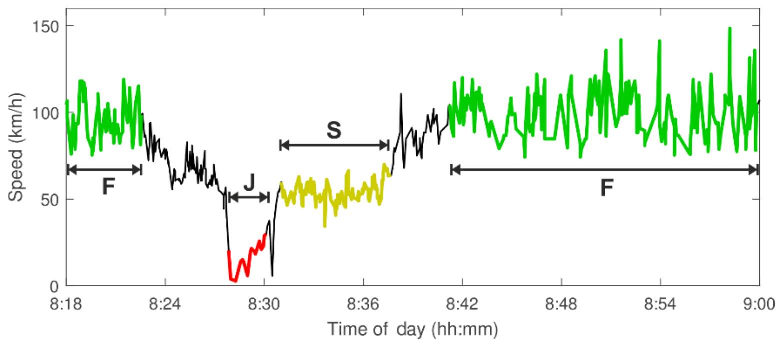


Fig. 4: Single vehicle data for speed within congested traffic measured at a detector on the German freeway A5. The data is separated into traffic phases: green is free flow (F), yellow is synchronized flow (S) and red is wide moving jam (J). The data marked in black is not considered because we cannot clearly assign a traffic phase to it.

In Tab. 1 the approximate number of measurements for each traffic phase is shown. Furthermore, the averaged time duration and the percentage of day for each traffic phase are included in Tab. 1.

Three traffic phases	Approximate number of measurements	Average time duration	Percentage of day
Free Flow	20 000	> 60 min	≈ 80 %
Synchronized Flow	4500	≈ 10 min	≈ 18 %
Jam	500	≈ 3 - 7 min	≈ 2 %

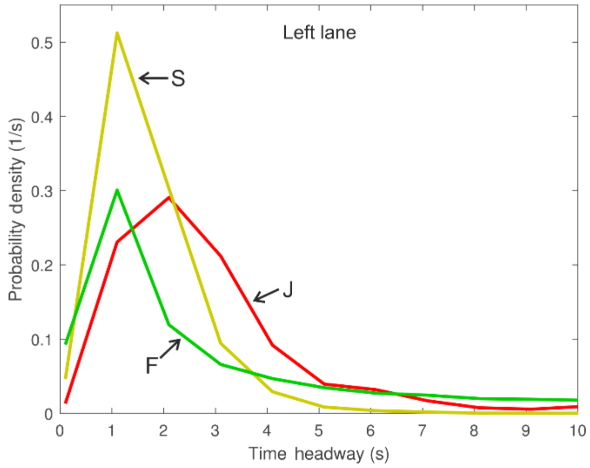
Tab. 1: Approximate number of measurements, averaged time length and averaged percentage of day for each traffic phase.

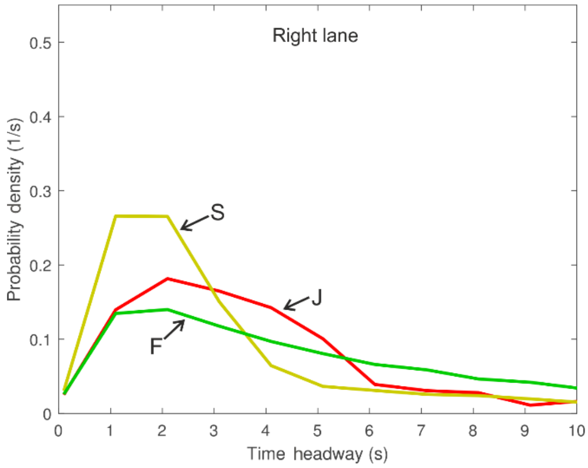
5 Analysis of Merging Opportunities

We aim to separate the time headway measurements into traffic phases. In Section 3 we described how to distinguish between all three traffic phases: free flow (F), synchronized flow (S) and wide moving jam (J).

5.1 Time Headway Analysis

We analyze the traffic phase of 25.000 time headway measurements and calculate the time headway distributions per lane and per traffic phase. Fig. 5 shows that the lowest time headways are found in synchronized traffic.





	Free	Sync	Jam
Mean	9.8 s	1.7 s	3.8 s
Median	2.5 s	1.5 s	2.5 s
P(Z > 4 s)	38 %	2 %	17 %

	Free	Sync	Jam
Mean	6.2 s	3.8 s	5.5 s
Median	4.5 s	2.3 s	3.5 s
P(Z > 4 s)	50 %	23 %	36 %

Fig. 5: Time headway distribution per traffic phase F, S and J (upper pictures) for left lane (upper left) and right lane (upper right). The calculated mean and median for each traffic phase are found in the bottom left table for the left lane and in the bottom right table for the right lane. The frequency of the time headway measurements, which are larger than 4 s, relating to each traffic phase are included in the tables (bottom pictures).

The time headway distributions in Fig. 5 support the following conclusions. Time headways strongly depend on the traffic phase. In synchronized flow the lowest time headways can be observed. Furthermore, time headways depend on the freeway lane. The median of the time headways for all three traffic phases on the left lane has a significant shift to a lower value than on the right lane. On the right lane free flow and wide moving jam show a similar time headway distribution. From Fig. 5 it can be concluded that the synchronized flow is the worst scenario for an automated vehicle in case of a merging maneuver.

5.2 Waiting Time Model and Analysis

In addition to the time headway distributions, we will investigate the waiting time of an automated vehicle, which is trying to merge from an on-ramp onto the freeway. The automated vehicle is only able to merge onto the freeway if there is a time gap that is equal or greater than the merging time threshold that the automated vehicle needs for a safe merging maneuver. Otherwise, it has to wait. We note that the needed time gap for merging describes an optimal case for which the automated vehicle can use this time gap. In many real cases other vehicles in flowing traffic might hinder the automated vehicles to merge. As discussed in Section 2, for automated vehicles we estimate the merging time threshold as 4 s for a comfortable and safe merging maneuver. We will also use other values for analysis purposes, e.g., 2 s, 4 s and 6 s.

We developed an analysis model based on the time headway data to investigate the expected waiting times for automated vehicles during a merging maneuver at freeway on-ramps. The approach is illustrated in Fig. 6.

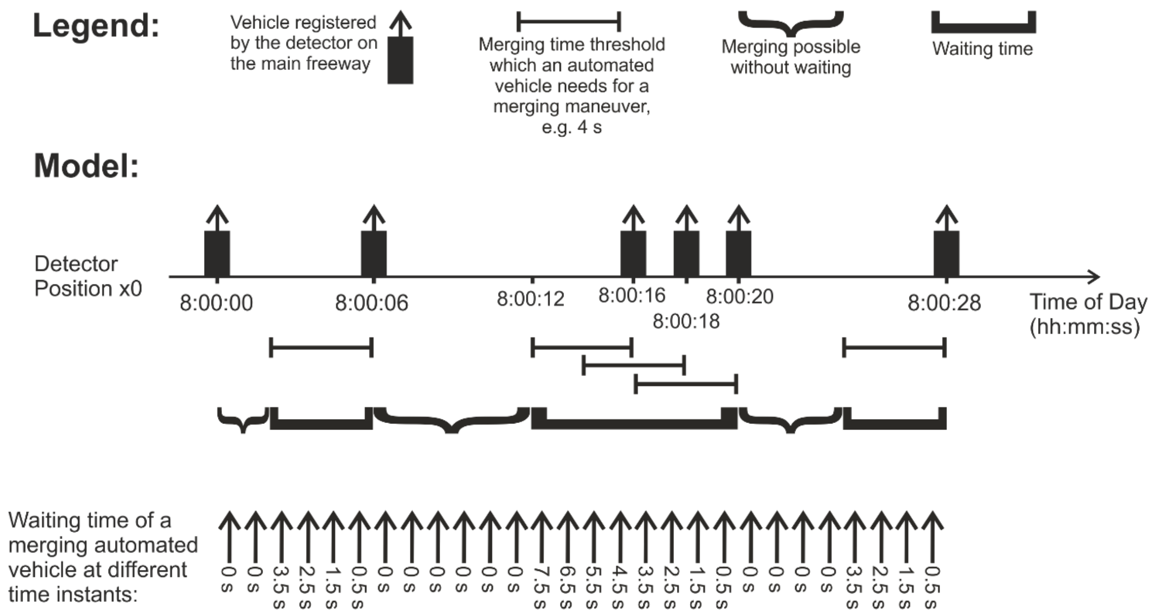
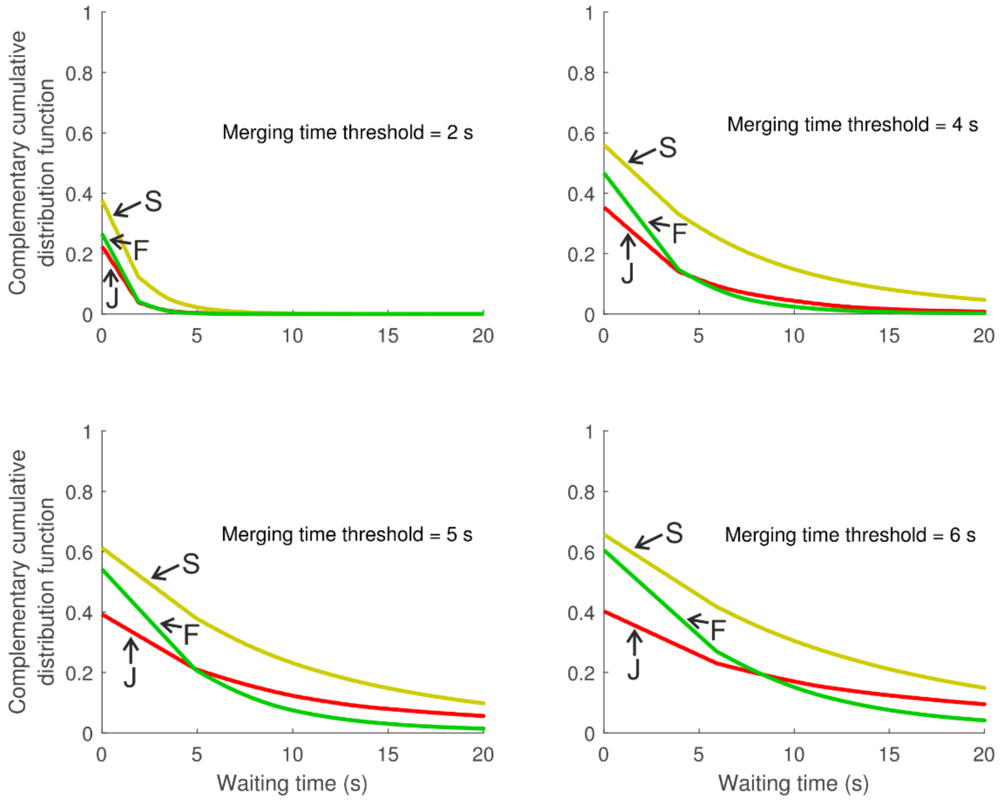


Fig. 6: Analysis model based on time headway data to investigate expected waiting times for automated vehicles before merging maneuvers at freeway on-ramps.

We know from time headway measurements the temporal distribution of these headways. Therefore, we can calculate the waiting time depending on the traffic phase under the assumption of different merging time thresholds. If the vehicles come always one after another with less than 4 s time headway, the waiting time will be infinite if the merging time threshold is 4 s. Fig. 7 shows the calculated results.



Waiting time for the right freeway lane

Merging time threshold	Mean			Median			Waiting time greater than 20 s		
	Free	Sync	Jam	Free	Sync	Jam	Free	Sync	Jam
6 s	4.7 s	9.3 s	5.7 s	1.9 s	3.9 s	0 s	4.2 %	15.0 %	9.5 %
5 s	3.0 s	7.0 s	3.7 s	0.7 s	2.4 s	0 s	1.4 %	9.8 %	5.6 %
4 s	1.7 s	4.4 s	1.7 s	0 s	1.1 s	0 s	0.2 %	4.7 %	0.8 %
2 s	0.4 s	0.7 s	0.3 s	0 s	0 s	0 s	0 %	0 %	0 %

Fig. 7: Complementary cumulative distribution functions and further statistics are shown for the waiting times for merging from a freeway on-ramp onto the right freeway lane. Different merging time thresholds are considered.

From Fig. 7 following conclusions can be made. First of all, we see that higher merging time thresholds from 2 – 6 s require higher waiting times. In particular, a three times higher merging time threshold increases the waiting time by a factor of more than 10. This shows that the waiting times of an automated vehicle are growing significantly faster than linearly with growing merging time thresholds. Furthermore, if we compare all three traffic phases (free flow (F), synchronized flow (S) and wide moving jam (J)) we see that longer waiting times and higher averaged waiting times occur in synchronized flow. The averaged waiting times are almost two times longer in synchronized flow than in free flow and in wide moving jam. Therefore, we can conclude that the waiting times are traffic phase dependent and that it is most difficult to merge into synchronized flow. For a merging time threshold of 6 s 15% of the waiting times in S are longer than 20 s, in F only 4.2% of the waiting times and in J 9.5% of the waiting times. Moreover, since averaged values in Fig. 7 are much higher than median values, we can conclude that very long waiting times might occur.

As mentioned an important conclusion is that the waiting times are growing significantly faster than linearly with growing merging time thresholds. It can also be concluded that the greater the merging time threshold, the greater the possibility that the automated vehicle will announce to the human driver to overtake the vehicle. For example, if the merging time threshold of an automated vehicle is 6 s, the waiting time is relatively often greater than 20 s in synchronized flow. If large waiting times, e.g., 20 s, can be predicted, the automated vehicle could announce to the human driver to overtake the vehicle to reduce the waiting time.

6 Conclusions

In this study, we analyzed measured time gap distributions of human drivers regarding different lanes and different traffic phases based on Kerner's three-phase traffic theory. The time gaps of human drivers strongly depend on the current traffic phase, e.g., in congested traffic with speeds up to 50 km/h on a freeway time gaps of less than 2 s often occur. Therefore, the time gaps and the related traffic phase of the non-automated vehicles on the main freeway influence the possibility of a proper merging maneuver of an automated vehicle at the on-ramp. If the possibility of a merging maneuver of an automated vehicle in a certain traffic phase is below a certain percentage, e.g., in S below 10%, the automated vehicle could announce to the human driver to overtake the merging maneuver because the automated vehicle is possibly unable to merge.

We illustrated an analysis of time headways of local sensor data and investigated the expected waiting times for automated vehicles during a merging maneuver at freeway on-ramps. We found that the merging possibility of automated vehicles is lane and traffic phase dependent. We discussed the merging possibility features in detail. Since the merging time threshold, which is the time an automated vehicle needs for a safe merging maneuver, is a crucial parameter of an automated vehicle, we used different values for the merging time threshold to get an insight of the impact of different merging time thresholds onto the waiting times. The intuitive expectation is confirmed: the greater the merging time thresholds, the greater the waiting times. However, the waiting times are growing significantly faster than linearly with growing merging time thresholds.

Traffic can be divided into three traffic phases: free flow, synchronized flow and wide moving jam [2-7]. As shown in this study, time gap information are traffic phase and lane dependent. Depending on the traffic phase and freeway lane, time gap information could be used as an input variable for a decision about a merging maneuver of an automated vehicle.

To get additional time headway and lane changing measurements for a road section with several lanes and observe this over a longer time interval, we plan in the scope of the project MEC-View [1] to use drones for further time gap measurements.

7 Acknowledgement

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